Technical Notes

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Approximate Analysis of Strake Wings at Low Speeds

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Introduction

THE analysis of strake wings, even at low speeds, is one of great complexity. The separated flow from highly swept leading edges of the strake influences the flow over the basic wing (having a prominent nose radius). While, in recent years, some advancements have been made for the analysis of highly swept wings having separated flow from the leading edges, and a number of well-established methods exist for the analysis of conventional wings having prominent nose radius, there are no suitable methods available for the analysis of strake wings when both types of flow are to exist simultaneously. The main problem seems to be to get the vortex field represented properly as it passes over the main wing. An approximate approach has been presented here. In this approach, slender wing theory is being used for the separated flow on the foward highly swept part (strake) and the upwash field from this (as a two-dimensional field) being fed into lifting surface theory.

The separated flow theory, as developed by Brown and Michael,[1] replaces the spiral vortex sheets from the straight leading edges of a highly swept wing by two concentrated vortices and two feeding vortex sheets connecting the leading edge and the concentrated line vortices. Their theory is applicable to delta wings having straight leading edges. Smith[2] extended the theory for delta wings having curved leading edges. The present investigation is restrictd to strakes having straight leading edges; thus, Brown and Michael's theory has been used for the analysis of the separated flow. The lifting surface theory used in the present treatment is the well-established vortex lattice theory.[3,4]

A FORTRAN-IV computer program has been developed to find the vortex strength distribution, chordwise and spanwise load distribution, and the overall characteristics of the wing. For the limited experimental results available, only the overall lift coefficient at different angles of attack have been compared and are shown in Figs. 1-3 for three examples taken from Refs. 5 and 6. The spanwise load distributions are shown in Fig. 4. The results of the present simplified approach appear to be highly convincing.

Mathematical Model

The problem considered here is an incompressible, inviscid flow past a flat plate at incidence α. The wing planform is a basic delta-type wing fitted with a thin straight-edged strake. The Cartesian coordinate system (x,y,z) has been considered originating at the leading edge of the center section, x axis along the chord, y axis along the starboard span, and z axis vertically upward. The flow is separated at the leading edges of the strake forming a spiral vortex sheet that influences the flow over the basic wing. The complexity of the flow structure has been simplified by combining Brown and Michael's method[1] with the vortex lattice method. Proper steps are taken for finding the downwash at each control point on the planform, due to the leading-edge vortex sheets approximated by a pair of isolated semi-infinite vortices formed on the strake and due to all the discrete horseshoe vortices representing the planform. The pair of isolated vortices will follow the path of the streamline, but it is assumed that these will remain parallel to the basic wing planform.

The simple formulas for the positions and strengths of the pair of isolated vortices at any chordwise position given by Brown and Michael[1] with the same symbols and notations are:

$$\frac{2\pi U_\infty \alpha}{\bar{\nu}} = \left(\frac{1}{\sqrt{\sigma_0^2+a^2}} + \frac{1}{\sqrt{\bar{\sigma}^2{}_0+a^2}}\right) \tag{1}$$

and

$$\frac{i\bar{\nu}}{2\pi}\left[\frac{\sigma_0}{(\sigma_0^2-a^2)+\sqrt{(\sigma_0^2-a^2)(\bar{\sigma}_0^2-a^2)}} - \frac{\sigma_0}{\sqrt{(\sigma_0^2-a^2)(\bar{\sigma}_0^2-a^2)}} - \frac{\sigma_0}{\sigma_0^2-a^2} + \frac{1}{2}\,\frac{a^2}{\sigma_0^2(\sigma_0^2-a^2)}\right]$$

$$= U_\infty \epsilon\left[2\frac{\bar{\sigma}_0}{a} - 1\right] \tag{2}$$

The real and imaginary parts of Eq. (2) give two equations for the unknown coordinates y_0 and z_0, which has been solved numerically along with Eq. (1).

Once we get the coordinates and strengths of the two isolated vortices (ξ_1,η_1,ζ_1), (ξ_2,η_2,ζ_2) and $\bar{\nu}$ at the trailing edge of the strake, the induced downwash w_s at any point (x,y,o) on the planform can be found out as:

$$w_s = \frac{\bar{\nu}}{4\pi}\left[\frac{y-\eta_1}{(y-\eta_1)^2-\zeta_1^2}\left(1+\frac{x-\xi_1}{[(x-\xi_1)^2+(y-\eta_1)^2+\zeta_1^2]^{1/2}}\right) - \frac{y-\eta_2}{(y-\eta_2)^2+\zeta_2^2}\left(1+\frac{x-\xi_2}{[(x-\xi_2)^2+(y-\eta_2)^2+\zeta_2^2]^{1/2}}\right)\right] \tag{3}$$

For the attached flow, the influence coefficient matrix A_{ij}, defined in Refs. 3 and 4 for the ith control point (x_i,y_i), due to the jth horseshoe vortex at the right wing and its counterpart on the left wing when multiplied by the horseshoe vortex strengths $\Delta\Gamma_j$ would give the upwash at all the control points. As shown by Giessing,[3] we are left with the problem of solving the matrix equation

$$[A_{ij}][\Delta\Gamma_j] = [W_{wj}] \tag{4}$$

where

$$W_{wj} = -U_\infty \alpha + w_s(x,y) \tag{5}$$

After getting the strengths $\Delta\Gamma_j$ over the wing planform, the chordwise pressure distribution ΔC_p can be calculated as:

$$\Delta C_p = -2\Delta\Gamma / U_\infty \Delta x \tag{6}$$

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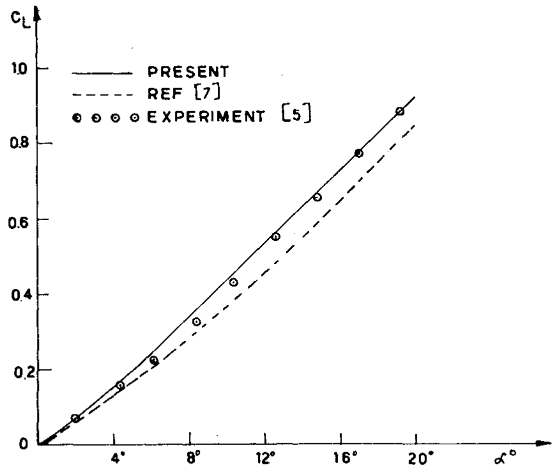

Fig. 1 Overall lift coefficients C_L vs angle of incidence for wing A.

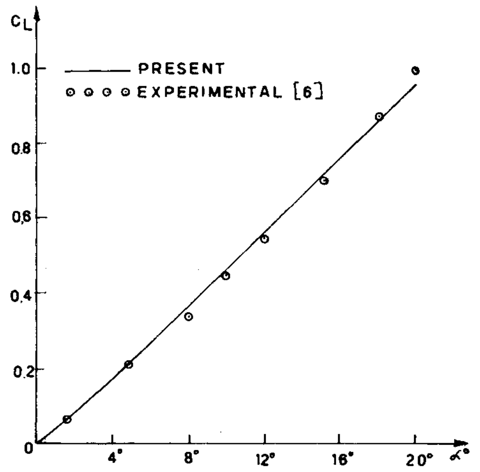

Fig. 3 Overall lift coefficients C_L vs angle of incidence for wing C.

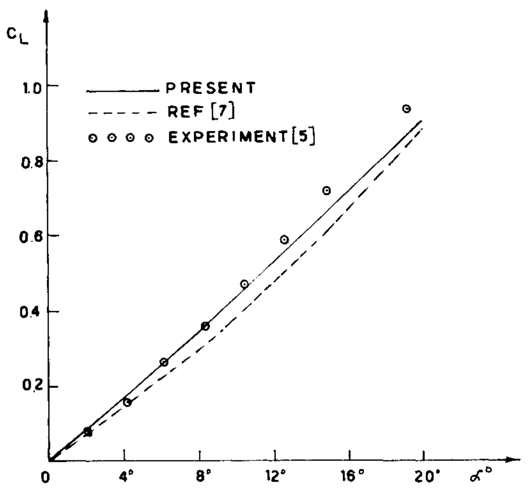

Fig. 2 Overall lift coefficients C_L vs angle of incidence for wing B.

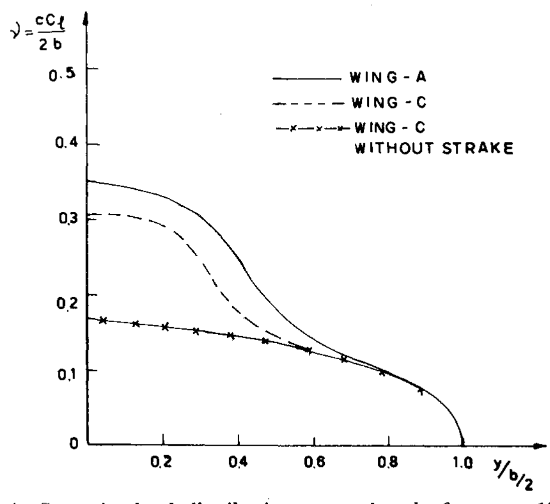

Fig. 4 Spanwise load distributions over the planforms at 12 deg incidence.

where Δx is the x length of the wing element over which $\Delta\Gamma$ acts. The local wing-lift coefficient C_l and the moment coefficient C_m are given by:

$$C_l = -\int_0^c \Delta C_p \frac{dx}{c} = \frac{2}{U_\infty c}\int_0^c \Delta\Gamma \approx \frac{2}{U_\infty c}\sum_j \Delta\Gamma_j \tag{7}$$

$$C_m = \int_0^C \Delta C_p \frac{x-x_m}{c}\frac{dx}{c} \approx -\frac{2}{U_\infty c^2}\sum_j (x_j - x_m)\Delta\Gamma_j \tag{8}$$

where c is the local chord length and x_m is the moment axis. The total wing lift and moment coefficients are:

$$C_L = 2\int_0^{b/2} cC_l \frac{dy}{A} \approx \frac{2}{A}\sum_k c_k C_{lk}\Delta y_k \tag{9}$$

$$C_M = 2\int_0^{b/2} C_m c^2 dy/(A\bar{c}) \approx \frac{2}{A\bar{c}}\sum_k C_{mk} c_k^2 \Delta y_k \tag{10}$$

where b is the span, A is the total wing reference area, and $\bar{c}$ is the reference chord length.

Numerical Examples

The examples considered here are taken from the experimental results.[5,6] The first model[5] consisted of a basic delta wing of 59 deg swept back, added with a strake of 77 deg swept. The larger and smaller strakes, wings A and B, correspond to aspect ratios 1.34 and 1.46, respectively. The trailing edge of the basic wing is 10 deg swept forward. The second model,[6] wing C, is a double-delta wing of 75 deg/62 deg swept with straight trailing edge of aspect ratio 1.61.

Taking account of the symmetry of the planform about the x axis, the semispan has been divided into twelve divisions and the chord into eight. By varying the number of divisions, it has been tested that the computed results do not vary much with the number of divisions and the 12×8 divisions give results of desired accuracy. Overall lift coefficient C_L of the planforms considered has been computed for different angles of attack and a comparison has been made with the experimental results. Figures 1-3 show the variation of C_L vs α for wings A, B, and C, respectively. Figures 1 and 2 show a better agreement of the present simple theory with the experimental results than the theory developed in Ref. 7. Figure 3 also shows an excellent agreement with experimental results. The nondimensional spanwise load distribution $\nu = cC_l/2b$ of wings A and C are also shown in Fig. 4.

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On the Calculation of Laminar and Turbulent Boundary Layers on Longitudinally Curved Surfaces

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Introduction

PREVIOUS contributions to the calculation of laminar boundary layers with longitudinal curvature have been made by Narasimha and Ojha,[1] van Tassel and Taulbee,[2] Schultz-Grunow and Breuer,[3] and Murphy[4]; earlier contributions have been reviewed by van Dyke.[5] Most authors have been concerned with isothermal flow (Ref. 2 is a major exception) and only Ref. 1 obtained detailed results for a range of wedge-flow pressure gradients. Turbulent boundary-layer properties have been calculated by So and Mellor,[6] Cebeci,[7] Bradshaw,[8] Rastogi and Whitelaw,[9] and Launder et al.[10] and have made use mainly of mixing-length forms of eddy viscosity. Once again, the emphasis has been on constant-property flow.

As indicated by Schultz-Grunow and Breuer,[3] the laminar boundary-layer equations with constant viscosity, including all second-order terms plus a diffusion term of higher order, may be written as:

$$u\frac{\partial u}{\partial x}+(1+ky)v\frac{\partial u}{\partial y}+kuv=-\frac{1}{\rho}\frac{\partial p}{\partial x}+(1+ky)\nu\frac{\partial^2 u}{\partial y^2}+\nu k\left[\frac{\partial u}{\partial y}-\frac{ku}{(1+ky)}\right] \tag{1}$$

$$ku^2=\frac{(1+ky)}{\rho}\frac{\partial p}{\partial y} \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial}{\partial y}(1+ky)v=0 \tag{3}$$

with

$$k=1/R_0$$

where the last term in brackets in Eq. (1) is of third order. Schultz-Grunow and Breuer[3] point out that a second-order treatment of the equations would omit this term, but provide small justification for the omission of other third-order terms which are present when boundary-layer scaling is used in the Navier-Stokes equations. Its inclusion simply allows the boundary condition, Eq. (4), to satisfy analytically the boundary-layer equations, Eqs. (1) and (2), at the "edge" $y=\delta$. By analogy, the same equations apply to turbulent boundary-layer flow with the laminar viscosity replaced by an effective viscosity $\nu+\epsilon_m$. For the present turbulent-flow calculations, the eddy viscosity (ϵ_m) was specified by the standard Cebeci-Smith formulation[11] modified by Bradshaw's correction for longitudinal curvature (see, for example, Ref. 11, Sec. 6.2.6) with a curvature parameter of 3.5.

Equations (1-3) are subject to the following boundary conditions:

$$y=0:\quad u=v=0;\quad y=\delta:\quad u_e=\frac{u_w(x)}{1+ky} \tag{4}$$

where the condition at $y=\delta$ has been obtained assuming $v=0$ and vanishing vorticity in the freestream, and $u_w(x)$ is the inviscid velocity distribution.

The boundary conditions at $y=\delta$ are slightly different from the usual edge boundary condition, namely $u=u_e$, because curvature also enters the condition at the outer edge of the boundary layer. For most boundary layers, transformed coordinates offer advantages and have been used here. By introducing the transformed variables (ξ,η) and a dimensionless stream function $f(\xi,\eta)$, Eqs. (1-4) can be written as:

$$(1+B\eta)(bf'')'+ff''+\frac{B}{1+B\eta}ff'-\frac{B^2}{(1+B\eta)}(bf')+2Bbf''-B(bf')'-\beta f'^2=P+2\xi\left[f'f'_\xi-\left(f''+\frac{B}{1+B\eta}f'\right)f_\xi\right] \tag{5}$$

$$\frac{B}{1+B\eta}f'^2=\frac{\partial p^*}{\partial\eta} \tag{6}$$

$$\eta=0:\quad f=f'=0;\quad \eta\rightarrow\eta_\infty:\quad f'\rightarrow 1/(1+B\eta) \tag{7}$$

Here, primes denote differentiation with respect to η and

$$B=\frac{k\sqrt{2\xi}}{\rho u_w},\quad \beta=\frac{2\xi}{u_w}\frac{du_w}{d\xi},\quad P=2\xi\frac{\partial p}{\partial\xi}$$

$$p^*=\frac{p}{\rho u_w^2},\quad b=1+\frac{\epsilon_m}{\nu}$$

Note that in Eq. (5) the term $B^2/(1+B\eta)(bf')$ corresponds to the higher-order viscous term of Eq. (1).

Unlike ordinary boundary-layer flows where $f'\rightarrow 1$ as $\eta\rightarrow\eta_\infty$, the edge boundary condition may be sensitive to the specification of η_∞. If we differentiate the edge boundary condition Eq. (7), then as $\eta\rightarrow\eta_\infty$,

$$f''\rightarrow -B/(1+B\eta)^2 \tag{8}$$

At first it may seem that the problem is overspecified. However, combining the edge condition, Eqs. (7) and (8), a relation for the outer boundary condition is obtained, namely,

$$\eta\rightarrow\eta_\infty\quad f''+B(f')^2=0 \tag{9}$$

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